

Rectangle Families

8th Grade Open Response Question

Academic Expectations:

1.5 Students use mathematical ideas and procedures to communicate, reason, and solve problems.

2.9 Students understand space and dimensionality concepts and use them appropriately and accurately.

Kentucky Core Content:

Concepts - Students will describe properties of, define, give examples of, and/or apply to both real-world and mathematical situations:

- **MA-M-1.1.3** *Meaning of proportion (equivalent ratios)*
- **MA-M-2.1.4** *Congruence, symmetry, and similarity*

Skills - Students will perform the following mathematical operations and/or procedures accurately and efficiently, and explain how they work in real-world and mathematical situations:

- **MA-M-4.2.3** *Model equations and inequalities concretely (e.g., algebra tiles or blocks), pictorially (e.g., graphs, tables), and abstractly (e.g., equations)*

Relationships - Students show connections and how connections are made between concepts and skills, explain why procedures work, and make generalizations about mathematics in meaningful ways for the following relationships:

- **MA-M-2.3.3** *How proportional figures are related (scale drawings, similar figures)*

THE PROBLEM

You work in a gift shop and have been asked, by your manager, to arrange a new shipment of picture frames into three different display groups, based on their shape. You find the 12 new frames are all rectangles with the following dimensions: (the first number is the width in inches and the second number is the height in inches) 6 X 10; 8 X 12; 4 X 6; 4 X 8; 3 X 5; 5 X 10; 2 X 3; 9 X 15; 3 X 6; 6 X 9; 1 X 2; 12 X 20.

- Arrange the frames into three groups/families, each group containing rectangles of similar shape.
- Explain how you know that each group contains only similar rectangles.
- On a separate sheet of grid paper, graph the dimensions of each frame/rectangle with the width recorded as the "x" variable and the height recorded as the "y" variable. For example, if the frame size is 6 inches wide and 10 inches high, you would place a dot at the coordinate (6,10). Connect all the points representing frames in a single group of similar rectangles. What do you notice about the shapes made by connecting the points in the 3 separate groups?
- Find the dimensions of an additional, similar frame size that would fit into each rectangle family. Explain your method.

Name _____

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- a) Arrange the frames into three groups/families, each group containing rectangles of similar shape.

Family A Dimensions	Family B Dimensions	Family C Dimensions

- b) Explain how you know that each group contains only similar rectangles.
- c) On a separate sheet of grid paper, graph the dimensions of each frame/rectangle with the width recorded as the “x” variable and the height recorded as the “y” variable. For example, if the frame size is 6 inches wide and 10 inches high, you would place a dot at the coordinate (6,10). Connect all the points representing frames in a single group of similar rectangles. What do you notice about the shapes made by connecting the points in the 3 separate groups?
- d) Find the dimensions of an additional, similar frame size that would fit into each rectangle family. Explain your method.

SCORING GUIDE

4	Student earns 4 points.
3	Student earns 3 points.
2	Student earns 2 points.
1	Student earns 1 point.
0	Student earns 0 points.

1 point—Part a) Student correctly lists the dimensions of rectangles for all three groups. *The heading label for each grouping is flexible. Families A, B, and C can contain any group of dimensions, as long as those dimensions are similar.*

1 point—Part b) Student thoroughly explains a method for proving that the rectangles in each family are similar.

Methods could include:

- Drawing the rectangles, one inside the other, and connecting the diagonals.
- Discussing the linear relationship found on the graph.
- Setting up proportions in which the sides of the rectangles are compared.
- Finding and simplifying the ratio of width/height for each rectangle.

1 point—Part c) Student correctly graphs three lines (see distinguished response, next page), each with 4 points corresponding to the dimensions of the rectangles.

0.5 points—Part d) Student correctly finds dimensions for a similar rectangle for each of the three categories.

0.5 points—Part d) Student thoroughly explains a method for finding the dimensions of a similar rectangle.

Methods could include:

- Finding the ratio of width/height and simplifying the fraction before multiplying by another factor.
- Extending the graph line to another intersection of integers to find another point on the line.

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Distinguished Response

THE PROBLEM

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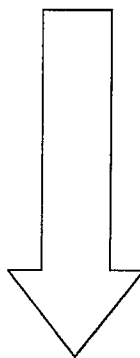
- a) Arrange the frames into three groups/families, each group containing rectangles of similar shape.

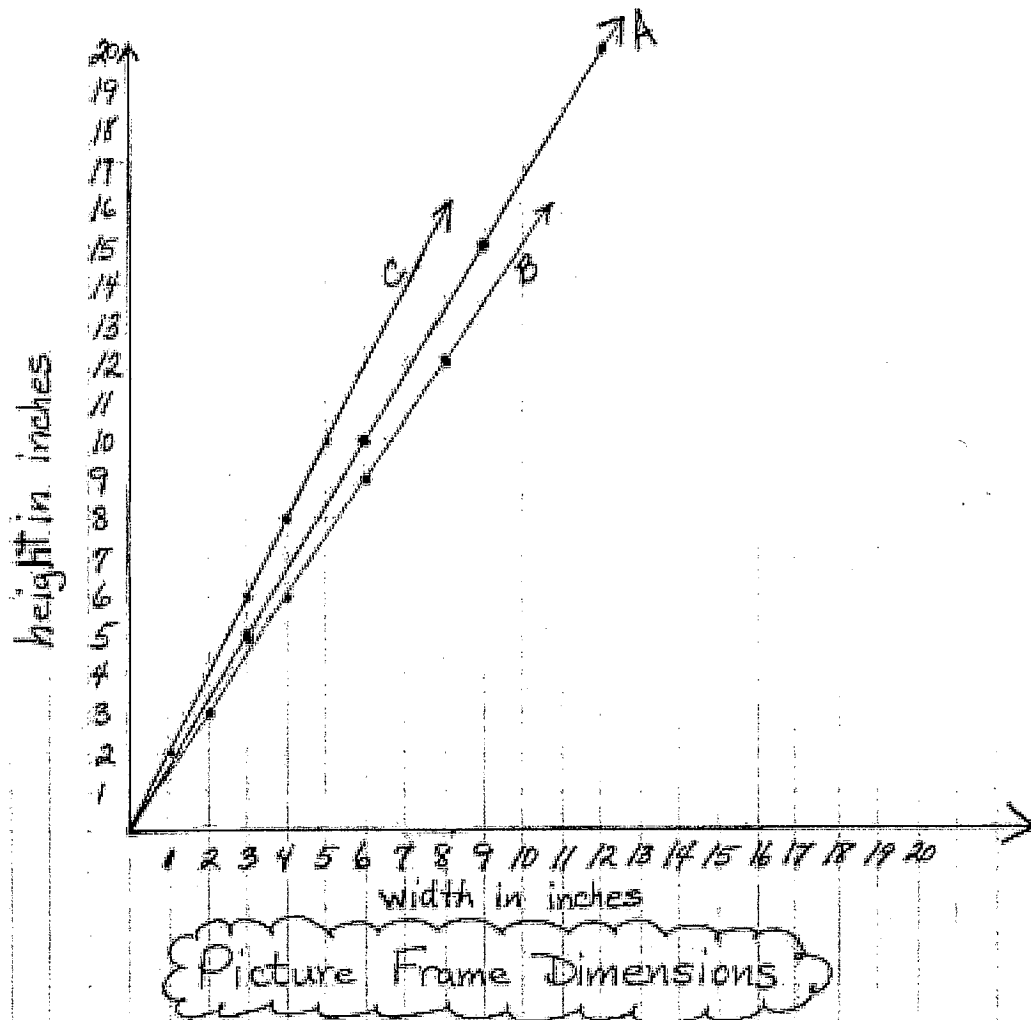
Family A Dimensions	Family B Dimensions	Family C Dimensions
6 X 10	8 X 12	4 X 8
3 X 5	4 X 6	5 X 10
9 X 15	2 X 3	3 X 6
12 X 20	6 X 9	1 X 2

- b) Explain how you know that each group contains only similar rectangles.

I know that each group contains only similar rectangles, because similar rectangles have the same ratio for width/height. If all the rectangles in a family are similar, then the ratios of width/height should give equivalent fractions. For family A: $6/10 = 3/5 = 9/15 = 12/20$. I know these fractions are all equivalent, because they can each be divided by the greatest common factor and simplified to $3/5$. The same is true for families B and C. $8/12 = 4/6 = 2/3 = 6/9$. $4/8 = 5/10 = 3/6 = 1/2$.

- c) On a separate sheet of grid paper, graph the dimensions of each frame/rectangle with the width recorded as the "x" variable and the height recorded as the "y" variable. For example, if the frame size is 6 inches wide and 10 inches high, you would place a dot at the coordinate (6,10). Connect all the points representing frames in a single group of similar rectangles. What do you notice about the shapes made by connecting the points in the 3 separate groups?





d) Find the dimensions of an additional, similar frame size that would fit into each rectangle family. Explain your method.

The additional, similar frame size that would fit into each family is as follows: family A—15 X 25; family B—10 X 30; family C—6 X 12. I found the size for each family by taking the simplified fraction width/height and multiplying it times a larger factor than the greatest common factor used in the largest picture frame. For example, I found the largest picture frame in family A, 12 X 20. Then I found the simplified fraction for width/height by dividing by 4, the greatest common factor $12/20 \div 4/4 = 3/5$. Finally, I took the simplified fraction and chose a factor greater than the greatest common factor, in this case a 5. The simplified fraction (width/height) times any factor will give a similar rectangle that fits into the family.